

# Membrane dynamics and matrix regularisation

Notes C Ross, September 20, 2017

These are notes for a talk I gave on the relationship between the quantisation of a classical super-membrane and matrix quantum mechanics. The talk was part of the seminar series which took place as a continuation of the strings and D-branes reading group<sup>1</sup>. These notes are mostly a rewriting of the material in [1] as that was the most accessible source. The talk was just on Section 2 and 3 with a small amount of motivation drawn from Section 1

## 1 Supergravity and some facts about strings and membranes

These are just some general facts about supergravity (Sugra) and string theory from the introductory section of [1].

Sugra is a theory with “local” (gauged) super symmetry. One way to see this is by looking at the anti-commutator of two (spinor) super charges,

$$\{Q, Q\} \sim P_\mu. \quad (1.1)$$

As this anti commutator is proportional to the momentum generator  $P_\mu$  which becomes a local vector field in GR generating a diffeomorphism.

The maximum dimension for Sugra is  $D = 11$ . If the dimension is greater than 11 there will be fields of spin greater than 2 in the theory and these are problematic to work with. **I think Weinberg QFT talks about this in detail. It is to do with defining a current associated with the field.**

There is a unique locally supersymmetric classical theory in  $D = 11$ . This theory has  $\mathcal{N} = 1$  so the  $Q$ 's live in a single 32-component spinor representation of the 11 dimensional Lorentz group. The theory has the following fields:

- $e_I^a$  the Vielbein, a 44 component bosonic field. This is an alternative description of the metric field  $g_{IJ}$ .
- $A_{IJK}$  the 3-form potential, an 84 component bosonic field.
- $\psi_I$  Majorana fermions (gravitino), a 128 component fermionic field.

Here we use  $I, J, K \in \{0, 1, \dots, 9, 11\}$  and  $\mu, \nu \in \{0, 1, \dots, 9\}$ . **The number of fermionic and bosonic components match up because of the supersymmetry.**

In ten dimensions there are two  $\mathcal{N} = 2$  theories with the  $Q$ 's as two 16-component spinors. The theories are

- IIA, the two spinors are of opposite chirality.

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<sup>1</sup>see [http://www.macs.hw.ac.uk/~cdr1/String\\_reading\\_group](http://www.macs.hw.ac.uk/~cdr1/String_reading_group) for details of the reading group and the other talks in the seminar series.

- IIB, the two spinors have the same chirality.

The field content of the two theories are

IIA:  $e_\mu^a, \phi, B_{\mu\nu}, C_\mu^{(1)}, C_{\mu\nu\rho}^{(3)}$ .

IIB:  $e_\mu^a, \phi, B_{\mu\nu}, C^{(0)}, C_{\mu\nu}^{(2)}, C_{\mu\nu\rho\sigma}^{(4)}$ .

Where  $\phi$  is a dilaton field and the  $C_{\mu_1\dots\mu_p}^{(p)}$  are Ramond-Ramond p-form fields. The  $B$  field couples to a string world sheet through

$$\int_{\Sigma} B_{\mu\nu} \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu, \quad (1.2)$$

where now  $a, b \in \{0, 1\}$  are related to the string world sheet coordinates. The string has two interesting, and related, quantities its length,  $l_s = \sqrt{\alpha'}$  and its tension  $T_s = \frac{1}{2\pi\alpha'}$ . The  $p + 1$  R-R field couples to a  $Dp$ -brane in an analogous manner. Fields could also couple magnetically but we will not give those details here.

We now note some facts about string theory:

1. World sheet superstring quantisation gives a first quantised theory of gravity from the point of view of the target space. This means that a state in the string Hilbert space corresponds to a single particle state in the target space of a single string.
2. The world sheet approach is perturbative in the string coupling  $g_s$ , related to the expectation of the dilaton field.

In  $D = 11$  SUGRA there is a ‘‘Black-membrane’’ solution analogous to a string, with world-volume  $\Sigma$ , which couples to the three form through

$$\int_{\Sigma} A_{IJK} \epsilon^{abc} \partial_a X^I \partial_b X^J \partial_c X^K, \quad (1.3)$$

where now  $a, b, c \in \{0, 1, 2\}$  give the membrane 3-volume coordinates. We can, and will later on, quantise the supermembrane in light cone coordinates to get a matrix quantum mechanics theory. This theory will have two advantages:

1. It provides a microscopic description of quantum gravity in 11 dimensions.
2. It provides a non-perturbative definition which is second quantised in the target space.

We can relate type *IIA* string theory to M-Theory through a duality. To go from M-theory in 11 dimensions to string theory we take the theory on  $M^{10} \times S^1$  where the circle has radius  $R$ . Taking  $R$  to be small we get *IIA*. Objects in the two theories are related as follows

- “wrapped” membrane goes to fundamental string,
- unwrapped membrane goes to  $D2$ -brane,
- “wrapped”  $M5$ -brane goes to  $D4$ -brane,
- unwrapped  $M5$ -brane goes to  $NS5$ -brane (magnetically charged under  $B$  field).

There is also a relationship between the constants on the string side and the 11 dimensional Planck length,  $l_{11}$  and  $R$  given by

$$g_s = \left( \frac{R}{l_{11}} \right)^{\frac{3}{2}}, \quad l_s^2 = \frac{l_{11}^3}{R}. \quad (1.4)$$

These relations suggest that when we want to go from type  $IIA$  string theory to M-theory if we take the limit  $g \rightarrow \infty$  then an extra dimension emerges to give M-theory in flat space. **I'm not going to pretend that I understand this relationship, I can sort of see the dimensional reduction direction but the emergent dimension is a puzzle.**

An application of this relationship is that if we consider the  $p_{11}$  momentum modes associated with the graviton multiplet then this corresponds to massive Kaluza-Klein particles in 10 dimensions coupled to  $g_{\mu 11}$  ( $C_\mu^1$ ). These particles correspond to  $D0$ -branes in  $IIA$  which gives us our first hint of the importance of the  $D0$ -brane gauge theory. As we saw in [2] The gauge theory on the world-volume of a  $D0$ -brane is a supersymmetric matrix quantum mechanics.

## 2 Classical bosonic membrane theory

We now turn our attention to a Polyakov-type approach to the world-volume theory of a classical bosonic membrane. Again we will be following the method given in [1].

Consider our membrane in flat  $D$ -dimensional Minkowski space

$$\mathcal{M}^D \simeq \mathbb{R}^{D-1,1}. \quad (2.1)$$

A dynamical membrane sweeps out a 3-dimensional world volume,  $\mathcal{V}$ , in  $\mathcal{M}^D$ . In other words the membrane is described by a map

$$X : \mathcal{V} \rightarrow \mathcal{M}^D. \quad (2.2)$$

We choose the coordinates,  $\sigma^\alpha$   $\alpha \in \{0, 1, 2\}$  on  $\mathcal{V}$ , usually we will write  $\sigma^0 = \tau$  for the time-like coordinate and  $\sigma^a$ , with  $a \in \{1, 2\}$ , for the space-like coordinates. The motion will be described by  $D$  functions,  $X^\mu(\sigma^0, \sigma^1, \sigma^2)$ .

As with the string it is natural to consider a Nambu-Goto type action,

$$S = -T \int d^3\sigma \sqrt{-\det h_{\alpha\beta}}, \quad (2.3)$$

where the membrane tension is

$$T = \frac{1}{(2\pi)^2 l_p^3} \quad (2.4)$$

and

$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu \quad (2.5)$$

is the pullback of the flat metric,  $\eta = \text{diag}(-, +, \dots, +)$ . As in the case of the string the presence of the square root makes this action tricky to work with. However, we can reformulate it in the Polyakov-esque form

$$S = -\frac{T}{2} \int d^3\sigma \sqrt{-\gamma} (\gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu - 1). \quad (2.6)$$

Here  $\gamma_{\alpha\beta}$  is an auxiliary metric on  $\mathcal{V}$  and  $\gamma = \det\gamma_{\alpha\beta}$ . There is a key difference between this action and the Polyakov action for the string, the presence of the factor of  $-1$  in the brackets. This is needed as now we do not have a scale invariant theory. **In [1] it is referred to as a ‘‘cosmological’’ term presumably this is because it looks like a Gauss-Bonnet term.**

We can vary the action, Equation (2.6), with respect to  $\gamma_{\alpha\beta}$  to find the equation of motion

$$\gamma_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu = h_{\alpha\beta}. \quad (2.7)$$

If we substitute this into Equation (2.6) and use that  $\gamma^{\alpha\beta} \gamma_{\alpha\beta} = \delta_\alpha^\alpha = 3$  we arrive at Equation (2.3). Thus demonstrating that the two actions are equivalent.

If we vary with respect to  $X^\mu$  then we arrive at the equation of motion

$$\partial_\alpha (\sqrt{-\gamma} \gamma^{\alpha\beta} \partial_\beta X^\mu) = 0. \quad (2.8)$$

If we were dealing with the string then this is the point that we would want to gauge fix  $\gamma$  before trying to quantise the action. However, for the membrane we have 6 independent metric components and only three diffeomorphism symmetries so we cannot completely fix  $\gamma$ . We can fix  $\gamma_{0\alpha}$ ! We do this by taking

$$\gamma_{0a} = 0, \quad \gamma_{00} = -\frac{4}{\nu^2} \bar{h} \equiv -\frac{4}{\nu^2} \det h_{ab}, \quad (2.9)$$

where  $\nu$  is an arbitrary constant. If we do this we cannot fix anymore components of  $\gamma$  and we can only enforce this gauge when  $\mathcal{V}$  has the specific form of  $\Sigma \times \mathbb{R}$ , where  $\Sigma$  is a Riemann surface.

After this gauge choice we can use the equation of motion for  $\gamma_{\alpha\beta}$ , Equation (2.7), to eliminate it from the action and arrive at

$$S = \frac{T\nu}{4} \int d^3\sigma \left( \dot{X}^\mu \dot{X}_\mu - \frac{4}{\nu^2} \bar{h} \right) \quad (2.10)$$

**Example 2.1.** *To see this note that the gauge fixing, Equation (2.9), gives us that*

$$\gamma = \det\gamma_{\alpha\beta} = -\frac{4}{\nu^2} \bar{h} \det\gamma_{ab}. \quad (2.11)$$

Now the equation of motion for  $\gamma$ , Equation (2.7), gives us that

$$\det\gamma_{ab} = \det h_{ab} = \bar{h}, \quad (2.12)$$

and all together we have that

$$\sqrt{-\gamma} = \frac{2}{\nu}\bar{h} \quad (2.13)$$

Next we need to consider the term in brackets

$$\gamma^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu - 1 = \gamma^{00}\dot{X}^\mu\dot{X} + \gamma^{ab}\partial_a X^\mu\partial_b X_\mu - 1, \quad (2.14)$$

$$= -\frac{\nu^2}{4\bar{h}}\dot{X}^\mu\dot{X} + \gamma^{ab}\gamma_{ab} - 1, \quad (2.15)$$

$$= -\frac{\nu^2}{4\bar{h}}\dot{X}^\mu\dot{X} + 1, \quad (2.16)$$

where we have used Equations (2.7) and (2.7) as well as the identity

$$\gamma^{ab}\gamma_{ab} = \delta_a^a = 2. \quad (2.17)$$

Putting this all together we have that

$$S = -\frac{T}{2} \int d^3\sigma \sqrt{-\gamma} (\gamma^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu - 1), \quad (2.18)$$

$$= -\frac{T}{2} \int d^3\sigma \frac{2}{\nu}\bar{h} \left( -\frac{\nu^2}{4\bar{h}}\dot{X}^\mu\dot{X} + 1 \right), \quad (2.19)$$

$$= \frac{T\nu}{4} \int d^3\sigma \left( \dot{X}^\mu\dot{X} - \frac{4}{\nu^2}\bar{h} \right), \quad (2.20)$$

which is the claimed result.

To make contact with our usual intuition from classical mechanics we can use the canonical (presumably the structure on the Riemann surface  $\Sigma$ ) Poisson bracket structure on the membrane to rewrite the action yet again. This structure is that at constant  $\tau$  the bracket is

$$\{f, g\} = \epsilon^{ab}\partial_a f\partial_b g. \quad (2.21)$$

There will be a symplectic form associated with this Poisson structure and we choose the coordinates  $\sigma$  such that with respect to this symplectic form the area of the Riemann surface is

$$\int d^2\sigma = 4\pi. \quad (2.22)$$

I think that we are actually working the other way round. The Riemann surface  $\Sigma$  is Kähler and thus has a Kähler form and we would “invert” this to get the Poisson bracket. Using this Poisson bracket we can rewrite the action as

$$S = \frac{T\nu}{4} \int d^3\sigma \left( \dot{X}^\mu\dot{X} - \frac{2}{\nu^2}\{X^\mu, X^\nu\}\{X_\mu, X_\nu\} \right), \quad (2.23)$$

and the  $X^\mu$  equation of motion becomes

$$\ddot{X}^\mu = \frac{4}{\nu^2} \partial_a (\bar{h} h^{ab} \partial_b X^\mu) = \frac{4}{\nu^2} \{ \{ X^\mu, X^\nu \}, X_\nu \}. \quad (2.24)$$

Checking this is left as an exercise to the reader/ listener.

As we have both, partially, gauge fixed  $\gamma_{\alpha\beta}$  and eliminated it from the action there are some constraints. These are

$$\dot{X}^\mu \dot{X}_\mu = -\frac{4}{\nu^2} \bar{h} = -\frac{2}{\nu^2} \{ X^\mu, X^\nu \} \{ X_\mu, X_\nu \}, \quad (2.25)$$

$$\dot{X}^\mu \partial_a X_\mu = 0, \quad (2.26)$$

$$\Rightarrow \{ \dot{X}^\mu, X_\mu \} = 0. \quad (2.27)$$

Check that the audience thinks that these are easy to see.

At this point we can observe that the (classical, bosonic) dynamical membrane is a constrained dynamical system given by Equations (2.23), (2.25), (2.27). This theory has the  $D$  functions  $X^\mu$  on  $\mathcal{V} \simeq \Sigma \times \mathbb{R}$  as its degrees of freedom. We should also note here that while the theory is currently covariant, if not manifestly so due to separating the timelike and spacelike coordinates, it is hard to quantise due to the constraints. This was also the case for the string. However, the constraints, Equations (2.25) and (2.27), are non-linear in this case so a covariant quantisation is more complicated than it was for the string.

## 2.1 Light-cone coordinates and a Hamiltonian

To proceed we will use light cone coordinates on  $\mathcal{M}^D$ ,

$$X^\pm = \frac{1}{\sqrt{2}} (X^0 \pm X^{D-1}), \quad (2.28)$$

and work in the light cone gauge

$$X^+(\tau, \sigma^1, \sigma^2) = \tau. \quad (2.29)$$

This enables us to solve the constraints as

$$\dot{X}^- = \frac{1}{2} \dot{X}^i \dot{X}^i + \frac{1}{\nu^2} \{ X^i, X^j \} \{ X^i, X^j \}, \quad (2.30)$$

$$\partial_a X^- = \dot{X}^i \partial_a X^i. \quad (2.31)$$

To move to the Hamiltonian picture we need to calculate the conjugate momenta,

$$P_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = \frac{T\nu}{2} \dot{X}^\mu. \quad (2.32)$$

We can then see that the total momentum in the + direction is

$$p^+ = \int d^2\sigma P^+ = \frac{T\nu}{2} \int d^2\sigma \dot{X}^+ = 2\pi\nu T. \quad (2.33)$$

performing the Legendre transform and integrating out  $\tau$  we get that the Hamiltonian is

$$H = \frac{T\nu}{4} \int d^2\sigma \left( \dot{X}^i \dot{X}^i + \frac{2}{\nu^2} \{X^i, X^j\} \{X^i, X^j\} \right), \quad (2.34)$$

and it is subject to the constraint

$$\{\dot{X}^i, X^i\} = 0 \quad (2.35)$$

on the coordinates transverse to the light cone. **Checking this is left as an exercise to the reader/ listener but it is essentially the same calculation as can be done for the string.**

There is still a residual invariance under time-dependent area preserving diffeomorphisms, this is because these do not change the symplectic form and thus will not change the Hamiltonian. Even in light cone coordinates this theory is still hard to quantise, compare to the string which is fairly easy to quantise in light cone coordinates. To get round this we will utilise a matrix regularisation of the theory. This regularisation will actually make sense of the arbitrary parameter  $\nu$  that I have so far left unexplained.

### 3 Matrix regularisation

As we are again following the discussion from [1] we will only give the details for when  $\Sigma = S^2$ . However, the same  $U(N)$  matrix theory arises on a Riemann surface of arbitrary genus as demonstrated in [3]. If we take  $\Sigma = S^2$  then the symplectic form will be  $SO(3)$  invariant and we will describe functions on the membrane in terms of Cartesian coordinates  $(\zeta_1, \zeta_2, \zeta_3)$  satisfying  $\zeta_1^2 + \zeta_2^2 + \zeta_3^2 = 1$ . The Poisson brackets for these functions is

$$\{\zeta_A, \zeta_B\} = \epsilon_{ABC} \zeta_C, \quad (3.1)$$

with  $A, B, C \in \{1, 2, 3\}$ , this is the same algebraic structure as the generators of  $SU(2)$ . We will there fore associate the coordinate functions with the generators of the  $N$ -dimensional representation of  $SU(2)$ . The normalisation factor  $\nu$  will be  $\nu = N$ . The correspondence is

$$\zeta_A \rightarrow \frac{2}{N} J_A, \quad (3.2)$$

with the  $J_A$  being the generators of the  $N$  dimensional representation of  $SU(2)$  which satisfy the commutation relations

$$-i[J_A, J_B] = \epsilon_{ABC} J_C. \quad (3.3)$$

We can use spherical harmonics to express any function on the membrane as

$$f(\zeta_1, \zeta_2, \zeta_3) = \sum_{l,m} c_{lm} Y_{lm}(\zeta_1, \zeta_2, \zeta_3). \quad (3.4)$$

The spherical harmonics will be polynomials in the coordinates

$$Y_{lm}(\zeta_1, \zeta_2, \zeta_3) = \sum t_{A_1 A_2 \dots A_l}^{(lm)} \zeta_{A_1} \dots \zeta_{A_l} \quad (3.5)$$

where the coefficients are symmetric and traceless. We can use the coordinate Matrix correspondence, Equation (3.2), to approximate the spherical harmonics by the matrices  $\mathbf{Y}_{lm}$  for  $l < N$ . These approximations are constructed as

$$Y_{lm}(\zeta_1, \zeta_2, \zeta_3) \rightarrow \mathbf{Y}_{lm} = \left(\frac{2}{N}\right)^l \sum t_{A_1 A_2 \dots A_l}^{(lm)} J_{A_1} \cdots J_{A_l}. \quad (3.6)$$

The restriction on  $l < N$  is because higher monomials in the  $J_A$  will not be linearly independent.

N.B we can see that the number of independent matrices is the same as the number of independent spherical harmonic coefficients as

$$N^2 = \sum_{l=0}^{N-1} (2l+1). \quad (3.7)$$

We can use this to construct matrix approximations of functions of the  $\zeta$  through

$$f(\zeta_1, \zeta_2, \zeta_3) \rightarrow F = \sum_{l < N, m} c_{lm} \mathbf{Y}_{lm}. \quad (3.8)$$

We can also replace the Poisson brackets by matrix commutators in the following way, which is very reminiscent of the minimal prescription approach to quantisation,

$$\{f, g\} \rightarrow -\frac{iN}{2}[F, G]. \quad (3.9)$$

Integrals over the membrane, that is the world-volume a fixed  $\tau$ , can also be replaced by a matrix trace

$$\frac{1}{4\pi} \int d^2\sigma f \rightarrow \frac{1}{N} \text{Tr} F. \quad (3.10)$$

We know that the Poisson bracket of two spherical harmonics is given by

$$\{Y_{lm}, Y_{l'm'}\} = g_{lm, l'm'}^{l''m''} Y_{l''m''}, \quad (3.11)$$

which means that we can write the commutator of the matrix approximations as

$$[\mathbf{Y}_{lm}, \mathbf{Y}_{l'm'}] = G_{lm, l'm'}^{l''m''} \mathbf{Y}_{l''m''}, \quad (3.12)$$

where it can be shown that

$$\lim_{N \rightarrow \infty} -\frac{iN}{2} G_{lm, l'm'}^{l''m''} = g_{lm, l'm'}^{l''m''}. \quad (3.13)$$

Using this it can be shown that for two functions  $f, g$  satisfying  $\{f, g\} = h$  that

$$\lim_{N \rightarrow \infty} \frac{1}{N} \text{Tr} F = \frac{1}{4\pi} \int d^2\sigma f, \quad (3.14)$$

$$\lim_{N \rightarrow \infty} \frac{1}{N} \text{Tr} \left( \left( -\frac{iN}{2} [F, G] - H \right) J \right) = 0, \quad (3.15)$$



where  $J$  is the matrix approximation of any smooth function  $j$  on  $S^2$ .

At this point we can quote our dictionary for converting between continuous quantities and their matrix regularisation

$$\zeta_A \leftrightarrow \frac{2}{N} J_A, \quad \{\cdot, \cdot\} \leftrightarrow -i \frac{N}{2} [\cdot, \cdot], \quad \frac{1}{4\pi} \int d^2\sigma \leftrightarrow \frac{1}{N} \text{Tr}. \quad (3.16)$$

Applying this dictionary to the Hamiltonian, Equation (2.34), we arrive at the regularised Hamiltonian

$$H = (2\pi T) \text{Tr} \left( \frac{1}{2} \dot{\mathbf{X}}^i \dot{\mathbf{X}}^i - \frac{1}{4} [\mathbf{X}^i, \mathbf{X}^j] [\mathbf{X}^i, \mathbf{X}^j] \right). \quad (3.17)$$

The regularised equation of motion is

$$\ddot{\mathbf{X}}^i + [[\mathbf{X}^i, \mathbf{X}^j], \mathbf{X}^j] = 0 \quad (3.18)$$

and the constraint, known as the Gauss constraint becomes

$$[\dot{\mathbf{X}}^i, \mathbf{X}^i] = 0. \quad (3.19)$$

As we know have a classical theory with a finite number of degrees of freedom we can quantise it. This model has  $N \times N$  matrix degrees of freedom and the matrices  $\mathbf{X}^i$  are in the adjoint representation of the symmetry group  $U(N)$ . In [1] the case of  $\mathcal{V} \simeq T^2 \times \mathbb{R}$  is also sketched but I will not mention that here. It can be argued that the regularised theory does not depend on the topology of  $\mathcal{V}$  as we arrive at the same Hamiltonian. **I probably won't comment more on this but there are more details in [1].**

### 3.1 In a general background

So far we have discussed the case of a membrane in a flat background,  $\mathcal{M}^D$ . However, the same story can be written down in a general background with metric,  $g_{\mu\nu}$  and 3-form potential  $A_{\mu\nu\rho}$ . The new metric modifies the field in the Nambu-Goto type action, Equation (2.3), in the following way

$$h_{\alpha\beta} = g_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu, \quad (3.20)$$

and adds a copy of Equation (1.3) to make the action become

$$S = -T \int d^3\sigma \left( \sqrt{-\det h_{\alpha\beta}} + 6 \dot{X}^\mu \partial_1 X^\nu \partial_2 X^\rho A_{\mu\nu\rho}(X) \right). \quad (3.21)$$

Again this can be interpreted in a Polyakov esque form as

$$S = -\frac{T}{2} \int d^3\sigma \left( \sqrt{-\gamma} (\gamma^{\alpha\beta} \partial_\alpha g_{\mu\nu} X^\mu \partial_\beta X^\nu - 1) + 12 \dot{X}^\mu \partial_1 X^\nu \partial_2 X^\rho A_{\mu\nu\rho}(X) \right). \quad (3.22)$$

The same gauge fixing story can then be applied and regularising this would result in a more general matrix model. **I won't talk about this but [1] does give a brief mention to this regularisation.**

### 3.2 Including fermions

Just as in String theory where we go to the superstring to get fermionic states we need to consider a super-membrane. These theories can only be constructed in certain dimensions  $D = 4, 5, 7, 11$  and the number of independent susy generators is different in each case, 2, 4, 8, 16. It is stated in [1] that theories in dimensions other than  $D = 11$  are believed to be problematic quantum mechanically so  $D = 11$  is the critical dimension for a super-membrane just as 10 was for the superstring. [1] is light on details here and cites the original paper [4] for the details. **Unfortunately this paper is not available through the HW subscriptions so I will not be sketching any of the details.**

The Hamiltonian gets in the light cone approach gets an extra fermionic term to become

$$H = \frac{T\nu}{4} \int d^2\sigma \left( \dot{X}^i \dot{X}^i + \frac{2}{\nu^2} \{X^i, X^j\} \{X^i, X^j\} - \frac{2}{\nu} \theta^T \gamma_i \{X^i, \theta\} \right), \quad (3.23)$$

where  $\gamma_i$  are the  $SO(9)$  gamma matrices in the 16 dimensional representation and  $\theta$  is a 16 component Majorana spinor of  $SO(9)$ . We can apply the matrix regularisation procedure to this Hamiltonian to arrive at

$$H = (2\pi T) \text{Tr} \left( \frac{1}{2} \dot{\mathbf{X}}^i \dot{\mathbf{X}}^i - \frac{1}{4} [\mathbf{X}^i, \mathbf{X}^j] [\mathbf{X}^i, \mathbf{X}^j] + \frac{1}{2} \theta^T \gamma_i [\mathbf{X}^i, \theta] \right). \quad (3.24)$$

## 4 BFSS conjecture

If time allows I will try and mention something about the M-theory as a matrix model conjecture from [5] though the discussion will follow that given in [1].

## References

- [1] W. Taylor, M(atr)ix theory: Matrix quantum mechanics as a fundamental theory. *Rev. Mod. Phys.* **73** (2001) 419.
- [2] Richard J. Szabo. *An Introduction to String Theory and D-Brane Dynamics (With Problems and Solutions)*. ICP, 2 edition, 2011.
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